LEIDENFROST POINT FOR SUBHEATED DROPS

Yu. A. Buevich and V. N. Mankevich

UDC 536.24:536.423

A theory of the heat-exchange crisis on a heated surface with drops evaporating on it is proposed. The critical temperature is found in the form of a function of the physical parameters, size, and initial subheating of the drops.

When a drop is placed on a horizontal solid surface heated to a temperature T_w which exceeds the boiling point T_s, two situations can occur: when $T_w < T_L$, the drop runs over the surface and boils on it; when $T_w > T_L$, a thin vapor interlayer separates the drop from the surface, i.e., the drop vaporizes in the spheroidal state [1]. Experimental values of the critical Leidenfrost temperature T_L corresponding to such a shift in regimes and heat-exchange crisis depend very heavily on the conditions of the experiment and the method of measurement; they vary from values exceeding T_s by only hundredths of a degree [2, 3] to values several times greater than T_s [3-5]. The latter fact is connected with the effect of diverse physical factors on the crisis and makes it necessary that a detailed theoretical study be mounted.

The existence of the spheroidal state of the vaporizing drop is naturally related to the establishment of a balance between the force of gravity, attracting the drop to the surface, and the force of excess pressure in the vapor interlayer, keeping the drop from approaching the surface. If the drop is located on a rough surface, then the spheroidal state is possible only when $h > \Delta$; when $h \approx \Delta$, the heat-exchange crisis begins, accompanied by running (spreading) of the drop over the surface. If the surface is perfectly smooth, then the deciding role in the advent of this crisis — a theory of which was presented in [6] — will be played by the dependence of the equilibrium temperature on the evaporation surface on the pressure in the vapor interlayer. However, as simple estimates for actual situations show, in both cases the value of $T_L - T_s$ usually turns out to be several orders lower than the empirically observed value — if it is assumed that the drop is in thermodynamic equilibrium with its vapor and no special measures are taken in the experiments to maintain this state (see, e.g., [2, 3]). It can be said that allowing for the effects of molecular dynamic and thermal slip of the vapor in the interlayer (the presence of velocity and temperature jumps at its boundaries) and the forces of dispersive molecular interaction between the solid and liquid surfaces can significantly increase the theoretical value of $T_L - T_s$, but this increase is quite inadequate to explain the available empirical data [2-5]. This has to do with the fact that the initial temperature T_0 of drops falling on a heated surface is usually below the saturation temperature, and realization of the spheroidal state requires that the entire drop — or at least its lower surface — be heated to this temperature before spreading of the drop begins.

Thus, there arises the very complex problem of the transient heating of drops. Solution of this problem requires simultaneous study of the hydrodynamics of the vapor interlayer, which is thinned under the influence of gravity and surface tension, and transfer of heat into the layer and into the drops, with allowance for the heat of phase transformation.

For simplicity, below we examine a drop on a rough surface with $\Delta > 10^{-4}$ cm, when we can ignore the effect of velocity and temperature jumps and the presence of dispersive interaction, as well as the dependence of the equilibrium state on the evaporation surface on the surface on the pressure in the interlayer. We also ignore radiative heat flow to the drop and its reverse effect on the local temperature of the surface underneath it, along with vaporization from the upper and lateral surfaces of the drop. (If necessary, all of these effects can be approximately accounted for by means of the considerations presented in [3, 4, 7].) We will also suppose that the vapor interlayer is plane-parallel, that the properties of the vapor in the layer are uniform, and that the phase transformation is insubstantial until the lower surface of the drop reaches the temperature T_s. This allows us to ignore the effect of heat absorption as a result of vaporization during heating of the drop.

Ural State University, Sverdlovsk. Moscow Institute of Chemical Engineering. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 6, pp. 949-953, June, 1983. Original article submitted March 22, 1982.

0022-0841/83/4406-0643$07.50

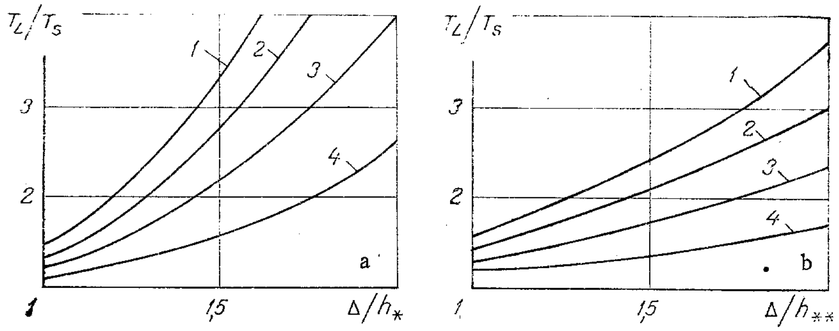

Fig. 1. Dependence of relative Leidenfrost point on Δ/h_* for uniformly heated drops (a) and on Δ/h_{**} for nonuniformly heated drops (b) with different drop subheatings: 1) $T_0/T_s = 0.2$; 2) 0.4; 3) 0.6; 4) 0.8.

The characteristic time of change in the thickness of the interlayer can be expressed as $h|dh/dt|^{-1}$. The characteristic times for establishment of a stationary temperature field and stationary velocity distribution in the layer are equal, respectively, to h^2/a'' and $\rho''h^2/\mu$. It is easily seen that with small h, the first time is significantly greater than the other two times (explicit estimates follow from the formulas presented below), i.e., the movement of the vapor and heat transfer in the interlayer can be examined in a quasistatic approximation.

The equations of mass and momentum conservation for an axisymmetric vapor flow in such an interlayer give us

$$u(x,\ r) = -\frac{1}{2\mu}\frac{dp}{dr}x(h-x),$$

$$2\pi r\int_0^h u(x,\ r)\,dx = -\frac{\pi r}{6\mu}h^3\frac{dp}{dr} = -\pi r^2\frac{dh}{dt}. \tag{1}$$

Regarding (1) as the equation for pressure, we obtain

$$p = p_R - \frac{3\mu}{h^3}\frac{dh}{dt}(R^2 - r^2). \tag{2}$$

Integrating (2) over the lower surface of the drop and equating the resulting pressure force to the force of gravity minus the buoyancy force, we arrive at the equation

$$\frac{dh}{dt} = -\frac{8}{9}\frac{R_0^3h^3}{\mu R^4}(\rho' - \rho'')g. \tag{3}$$

Inertial forces were not considered in deriving (3), since the acceleration $|d^2h/dt^2|$ is usually much less than g.

Given the condition $h(0) = h_0$, the solution of Eq. (3) has the form

$$t = \frac{9}{16}\frac{\mu R^4}{(\rho' - \rho'')gR_0^3}\left(\frac{1}{h^2} - \frac{1}{h_0^2}\right).$$

Assuming that h_0 is much greater than the characteristic values of h at which drop heating actually occurs, we obtain a relation connecting h and t and describing the dynamics of thinning of the vapor interlayer:

$$h \approx \frac{3}{4}\left[\frac{\mu R^4}{(\rho' - \rho'')gR_0^3t}\right]^{1/2}. \tag{4}$$

To analyze the change in the temperature T^* of the lower surface of the drop over time, it is necessary to solve a transient problem of heat conduction in a drop of the specified form. Here we assign the quasisteady heat flux $q = \lambda''(T_w - T^*)/h$ on the lower surface of the drop and specify heat exchange with the surrounding vapor or gas—vapor medium, with a temperature T_s, on the upper and lateral surfaces. In the general case, the solution of such a problem is very complicated and awkward, but it is not needed to explain the principal features of the process. Here we are primarily accounting for the fact that heat is transferred mainly through the lower surface of the drop, and we limit ourselves to analysis of only two limiting situations. In the first, where the characteristic time of temperature equalization inside the drop R_0^2/a' is much shorter than the characteristic heating time (the corresponding estimate the explicit form is also easily found on the basis of the results presented below), the temperature of the drop at any moment of time can be roughly assumed to be uniform. In the second situation, corresponding to satisfaction of the inverse force inequality at these moments, only the region of the drop immediately bordering its lower surface is important, i.e., drop heating can be regarded as the heating of an infinite half-space.

In the first case the kinetics of heating are described by the equation

$$\frac{4}{3}\pi R_0^3\rho'c'\frac{dT^*}{dt}=\frac{\pi R^2\lambda''}{h}(T_w-T^*),$$

the solution of which, with the condition $T^*(0) = T_0$, has the form

$$T^*=T_w-(T_w-T_0)\exp\left(-\frac{3\lambda''R^2}{4\rho'c'R_0^3}\int\limits_0^t\frac{dt}{h}\right).$$

The integral in the exponent is easily calculated by replacing the integration variable t by h in accordance with (3). We obtain

$$T^*=T_w-(T_w-T_0)\exp(-h_*^3/h^3),$$
$$h_*=\left[\frac{9}{32}\,\frac{\mu\lambda''}{\rho'c'(\rho'-\rho'')g}\right]^{1/3}\left(\frac{R}{R_0}\right)^2. \tag{5}$$

The Leidenfrost point is determined as the temperature T_w at which h is exactly equal to the height of the projections of the surface roughness Δ. Then from (5) we obtain ($T^* = T_s$)

$$\frac{T_L}{T_s}=\frac{1-(T_0/T_s)\varphi}{1-\varphi},\quad\varphi=\exp\left(-\frac{h_*^3}{\Delta^3}\right). \tag{6}$$

The dependence of T_L/T_s on Δ/h_* for different relative initial drop temperatures is shown in Fig. 1a.

In the second case we investigate the heat conduction equation in the half-space x > 0 with the initial and boundary conditions

$$-\frac{\partial T}{\partial x}=\frac{\lambda''}{\lambda'}\,\frac{T_w-T}{h}\bigg|_{x=0},\quad T\to T_0|_{x\to\infty},\quad T=T_0|_{t=0}.$$

With a sufficiency slow change in h over time, the stated problem reduces to that examined in [8]. The temperature T^* on the lower surface of the drop is equal to the following in this case:

$$T^*=T_0+(T_w-T_0)\left[1-\exp\left(\frac{\lambda''^2}{\lambda'^2}\,\frac{a't}{h^2}\right)\operatorname{erfc}\left(\frac{\lambda''}{\lambda'}\,\frac{\sqrt{a't}}{h}\right)\right]. \tag{7}$$

We again define the Leidenfrost point as that temperature T_w at which h coincides with Δ at the moment when T^* equals T_s. Allowing for (4), we obtain the following from (7):

$$\frac{T_L}{T_s}=\frac{1-(T_0/T_s)\varphi}{1-\varphi},\quad\varphi=\exp\left(\frac{h_{**}^4}{\Delta^4}\right)\operatorname{erfc}\left(\frac{h_{**}^2}{\Delta^2}\right),$$
$$h_{**}=\left(\frac{3}{4}\right)^{1/2}\left[\frac{\lambda''}{\lambda'}\,\frac{\mu\lambda''R_0}{\rho'c'(\rho'-\rho'')g}\right]^{1/4}\frac{R}{R_0}. \tag{8}$$

The dependence of T_L/T_s on φ in (8) is similar to that in (6), but the expression for φ in terms of the physical parameters and Δ is quite different. The dependence of T_L/T_s on Δ/h_{**} for different relative initial subheatings of the drop is shown in Fig. 1b.

It can easily be seen that the curves in Fig. 1b are similar to those in Fig. 1a and correspond to roughly the same type of dependence of the Leidenfrost point on the physical parameters. The dependence of T_L/T_s on the ratio R/R_0 is modified most strongly with an increase in the thermal conductivity of the liquid (with a gradual transition from the second of the situations being examined to the first): whereas h_{**} is proportional to the first power of this ratio, h_* is proportional to its square. The value or R/R_0 can be obtained by solving the problem of the static equilibrium of a drop with surface tension at its boundary in a gravitational field. This problem has been studied repeatedly. In particular, the following asymptotes are valid for coarse and fine drops [3]:

$$\frac{R}{R_0} \approx \left(\frac{2R_0}{3\varkappa}\right)^{1/2}, \quad R_0 \gg \left[\frac{\sigma}{(\rho' - \rho'')g}\right]^{1/2} = \varkappa,$$

$$\frac{R}{R_0} \approx \left(\frac{2}{3}\right)^{1/2} \frac{R_0}{\varkappa}, \quad R_0 \ll \varkappa.$$

Above we did not consider the intensive internal circulation characteristic of drops in the spheroidal state. It is obvious that convective heat transfer inside the drop makes it appreciably more probable that its temperature will be equalized.

When $T_0 \to T_s$, it follows from the results presented above that $T_L \to T_s$, i.e., within the framework of the model examined here, the Leidenfrost point is determined entirely by the initial subheating of the drops. This is understandable from a physical point of view, since above, for simplicity, we ignored molecular slip at the boundaries of the vapor interlayer, forces of dispersive interaction between the solid and liquid surfaces, and the dependence of the equilibrium temperature on the vaporization surface on the pressure close to this surface.

NOTATION

a', a'', diffusivity of the liquid and vapor; c', specific heat of the liquid; g, acceleration due to gravity; h_*, h_{**}, characteristic thicknesses of vapor interlayer h; p, p_R, pressure in the interlayer and on its outer boundary; q, heat flux; R, R_0, radii of vapor interlayer and spherical drop; r, radial coordinate; T_L, T_W, T_s, T^*, T_0, Leidenfrost point, temperature of heated wall, saturation temperature, temperature of lower surface of drop, and initial temperature of drop, respectively; t, time; x, normal coordinate; Δ, height of surface roughness; λ', λ'', thermal conductivity of liquid and vapor; μ, viscosity of vapor; ρ', ρ'', density of liquid and vapor; φ, parameter introduced in (6) and (8); $\varkappa$, capillary constant; σ, surface tension.

LITERATURE CITED

1. S. S. Kutateladze and M. A. Styrikovich, Hydrodynamics of Gas—Liquid Systems [in Russian], Énergiya, Moscow (1976).
2. E. S. Godleski and K. J. Bell, "The Leidenfrost phenomenon for binary liquid solutions," in: Proc. 3rd Int. Heat Transf. Conf., Vol. 4, AIChE, New York (1966), pp. 51-58.
3. L. H. J. Wachters, H. Bonne, and H. J. van Nouhuis, "The heat transfer from a hot horizontal plate to sessile water drops," Chem. Eng. Sci., 21, No. 8, 923-936 (1966).
4. K. J. Baumeister and F. F. Simon, "Leidenfrost temperature — its correlation for liquid metals, cryogens, hydrocarbons, and water," Trans. ASME J. Heat Transfer, 95C, No. 1, 166-173 (1973).
5. I. Michiyoshi and K. Makino, "Heat transfer characteristics of evaporation of liquid of liquid droplets on heated surfaces," J. Heat Mass Transfer, 21, No. 3, 605-613 (1978).
6. Yu. A. Buevich and V. N. Mankevich, "Theory of liquid vaporization in the spheroidal state," Dokl. Akad. Nauk SSSR, 262, No. 6, 1373-1377 (1982).
7. K. J. Baumeister, Th. D. Hamill, and G. J. Schoessow, "A generalized correlation of vaporization times of drops in film boiling on a flat plate," in: Proc. 3rd Int. Heat Transf. Conf., Vol. 4, AIChE, New York (1966), pp. 66-73.
8. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).